

Unique Paper Code : 62351101_OC
Name of the Paper : Calculus
Name of the Course : B.A. (Prog.) I Year
Semester : I
Duration : 3 Hours
Maximum Marks : 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Find the limit of the given functions

$$(i) \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^2 \sin x} \quad (ii) \lim_{x \rightarrow 0} \frac{1}{1 + e^{1/x}}$$

Discuss the continuity and discontinuity of following functions,

$$(i) f(x) = |x - 1| + |x - 2| \text{ at } x = 1 \text{ \& } x = 2$$

$$(ii) f(x) = \begin{cases} \frac{x e^{1/x}}{1 + e^{1/x}}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases} \text{ at } x = 0.$$

Can a function have more than one limit? Explain.

2. If $y = \sin mx + \cos mx$, then show that $y_n = m^n [1 + (-1)^n \sin 2mx]^{1/2}$.

State Leibnitz's theorem. If $y = \sin^{-1} x$ then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$

If $u = x^2 \tan^{-1} \left(\frac{x}{y} \right) - y^2 \tan^{-1} \left(\frac{y}{x} \right)$ then prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

3. Prove that the curve $\left(\frac{x}{a} \right)^n + \left(\frac{y}{b} \right)^n = 2021$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b) , whatever be the value of n .

Show that normal at any point of the curve

$$x = a \cos \theta + a \theta \sin \theta, y = a \sin \theta - a \theta \cos \theta$$

is at constant distance from origin.

Find the curvature at any point θ on the curve

$$y = a \left(\cos \theta + \log \left(\tan \left(\frac{\theta}{2} \right) \right) \right), y = a \sin \theta.$$

4. Find asymptotes of the curve $(y - 3)^2(x^2 - 9) = x^4 + 81$.

Determine the position and nature of double points on the curve

$$x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0$$

Trace the curve $x^2y^2 = x^2 - 25a^2$.

5. State Lagrange's mean value theorem and give its geometrical interpretation. Applying Lagrange's mean value to the function defined by $f(x) = \log(1 + x) \forall x > 0$ show that

$$0 < [\log(1 + x)]^{-1} - x^{-1} < 1 \text{ whenever } x > 0.$$

Separate the interval in which the function defined on \mathbb{R}

$$f(x) = 2x^3 - 15x^2 + 36x - 1$$

is increasing or decreasing.

Prove that $\tan x > x$ whenever $0 < x < \pi/2$.

6. Obtain Maclaurin's series expansion of $\cos 2x$.

Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x \sin x}$

Find the maximum value of $\left(\frac{1}{x}\right)^x$.

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